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Tetrahedral Underwater Arrays and the Effect of Current on Their Shape

Ronald I. Verrall

DEFENCE RESEARCH ESTABLISHMENT ATLANTIC

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Abstract

Underwater acoustic arrays shaped like tetrahedrons are possible candidates for rapidly deployable systems (RDS). They have aperture in all three physical dimensions, and therefore might be suitable for matched-field processing. This paper calculates the deformations of such an array in a current and compares this deformation to that of a simple vertical array. The paper derives the equations of the cable shape and presents the results of a number of numerical integrations. It appears that the tetrahedral shapes are not as stiff as one would infer from their triangular nature. The paper then points out very simple equations that will predict the cable shapes under quite common conditions. These back-of-the-envelope equations may well be a very useful design tool.

Résumé

Des réseaux acoustiques sous-marins en forme de tétraèdres sont des candidats possibles de systèmes à déploiement rapide (SDR). Ces réseaux s'étendent dans les trois dimensions et peuvent donc convenir au traitement par champ de concordance. L'article présente les calculs des déformations subies par un tel réseau dans un courant et les compare à la déformation subie par un simple réseau vertical. On y définit aussi les équations relatives à la forme du câble et on y présente les résultats d'un certain nombre d'intégrations numériques. Il semble que les formes tétraédriques ne sont pas aussi rigides que leur nature triangulaire le laisse supposer. L'article signale ensuite des équations très simples qui permettent de prévoir les formes du câble dans des conditions assez courantes. Ces équations de premier jet pourraient bien constituer un outil de conception d'une grande utilité.

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Executive summary

INTRODUCTION:

The concept of a rapidly deployable system (RDS) is an important topic in today's underwater acoustic community. Much attention has been focussed on acoustic arrays that are quick and easy to install and that can get the acoustic data back to a central location in a timely manner. This array should preferably have spatial aspect in all three physical dimensions in order to make it amenable to matched-field processing as well as to conventional beamforming.

One suggestion has been to build an array having the shape of a tetrahedron. It would consist of three linear arrays joined together and buoyed at their upper ends. Their lower ends would be anchored on the bottom at the three corners of a triangle. The structure gives aperture in all three spatial dimensions, and the geometry, being composed of several triangles, gives the impression of being quite stiff. It looks as if it might not be as vulnerable to current drag as a simple vertical array. However, a counter argument points out that the edges of this structure are not rigid, and they will not even resist a bending moment - let alone compression.

This study was initiated to look into the resistance of a tetrahedral array to the effects of water currents. A useful offshoot has been the realization that under quite reasonable conditions the shapes and offsets of both tetrahedral arrays and vertical line arrays (and, by extension, any line array) can be expressed by very simple formulas. This constitutes a very useful design tool.

SUMMARY OF DISCUSSION:

The paper begins by deriving the equations that describe the shape of a cable hanging in a current. These equations derive from the statement that an element of the cable is in equilibrium under the forces acting upon it, namely the force of gravity, the drag of the current and the tension forces in the cable itself. In practice one starts the integration of these differential equations at a float, for example, where the tension force and its direction can be calculated. One then steps down the cable applying the differential equations to each new cable element - thus calculating the shape of the cable and the tension in it.

The paper then shows how to handle the problem when three cables are joined at the same float, in which case neither their tensions nor their orientations are directly calculable.

Numerical computations carried out on tetrahedral-shaped arrays indicate that the current does not move the float very much, and the downward motion is appreciably greater than the horizontal motion. However, although the float is reasonably 'rigid', the centre of each leg of the tetrahedron is pushed sideways by the current.

This deformation was then compared with that of a simple vertical line array - a cable having a weight at one end and a float at the other. The maximum deformation of the tetrahedral array was not appreciably less than that of the line array, and it was even somewhat greater when the flotation given to all three of the tetrahedral legs was applied to the single vertical array.

Although this stiffness is somewhat less than had been hoped, it is still good enough for the tetrahedral array to be considered as a candidate for an RDS.

Another significant result of this study was the observation that for neutrally buoyant arrays of either type, the shape of the array legs and their maximum offsets could be expressed as simple equations that can be evaluated with hand calculators.

SIGNIFICANCE OF RESULTS:

These results are quite useful for the design of rapidly deployable underwater systems, which are of appreciable concern to modern navies. The computer programs that solve the general equation will calculate the shapes of array cables that are not neutrally buoyant, and simple equations can calculate the shapes when the cables are neutrally buoyant, which, in practice, is most of the time. In the specific case of the tetrahedral array, calculations of its deformation in a current provide only moderate support for this shape.

FUTURE PLANS:

The technology demonstrator (TD) of the joint rapidly deployable system (JRDS) is about to get under way at DREA. Much work will be done to design underwater arrays that are suitable for the purpose. The computer codes and the simplified equations will be quite useful for this task.

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Sommaire

INTRODUCTION :

Le concept d'un système à déploiement rapide (SDR) est actuellement un sujet de discussion important dans le domaine de l'acoustique sous-marine. On prête beaucoup d'attention aux réseaux acoustiques qui peuvent être installés rapidement et facilement et qui peuvent transmettre les données acoustiques en temps opportun en un lieu central. Ce réseau devrait de préférence s'étendre dans les trois dimensions afin de le rendre sujet au traitement par champ de concordance ainsi qu'à la méthode classique de mise en forme du faisceau.
Une des méthodes proposées consiste à construire un réseau ayant la forme d'un tétraèdre. Celui-ci se composerait de trois réseaux linéaires reliés entre eux à leurs extrémités supérieures, ce point commun étant un flotteur. Leurs extrémités inférieures seraient ancrées au fond marin, aux trois sommets d'un triangle. La structure s'étend dans les trois dimensions et sa géométrie, composée de plusieurs triangles, semble très rigide. L'ensemble donne l'impression de beaucoup mieux résister à l'effet d'entraînement du courant qu'un simple réseau vertical. Par contre, un autre argument fait valoir que les arêtes de cette structure ne sont pas rigides et qu'elles ne résisteront même pas à un moment de flexion et encore moins à la compression.
Nous avons entrepris la présente étude pour déterminer le degré de résistance d'un réseau tétraédrique aux effets des courants marins. Les travaux ont mené à une retombée utile, soit la réalisation que, dans des conditions assez acceptables, on peut exprimer, à l'aide de formules très simples, les formes et les décalages des réseaux tétraédriques, tout comme ceux des réseaux linéaires verticaux (et, par extrapolation, de tout réseau linéaire). L'ensemble constitue un outil de conception d'une grande utilité.

SOMMAIRE DE LA DISCUSSION :

Dans la première partie de l'article, nous établissons les équations qui décrivent la forme d'un câble suspendu dans un courant. Ces équations découlent de l'affirmation selon laquelle un élément du câble est en équilibre sous les forces qu'il subit, nommément la pesanteur, l'effet d'entraînement du courant et les tension présentes dans le câble même. Habituellement, l'intégration de ces équations différentielles s'amorce à un point, par exemple un flotteur, où on peut calculer la tension et sa direction. On descend ensuite graduellement le long du câble en appliquant les équations différentielles à chaque nouvel élément de ce dernier et on peut ainsi calculer la forme du câble et la tension dans celui-ci.
La partie subséquente de l'article concerne la résolution du problème constitué de trois câbles fixés à un même flotteur, un cas pour lequel on ne peut calculer directement les tensions et les directions respectives des câbles.
Les résultats du traitement numérique des réseaux de forme tétraédrique indiquent que le courant ne déplace pas beaucoup le flotteur et que le déplacement descendant est sensiblement plus important que le déplacement horizontal. Cependant, bien que le flotteur soit assez « rigide », le point milieu de chaque bras du tétraèdre subit une poussée latérale exercée par le courant.
Nous avons ensuite comparé cette déformation à celle subie par un simple réseau linéaire vertical, c'est-à-dire un câble auquel sont fixés un poids à l'une de ses extrémités et un flotteur à la seconde. La déformation maximale du réseau tétraédrique n'est pas beaucoup moins importante que celle du réseau linéaire et elle est même légèrement supérieure, dans le

cas précis où la valeur du paramètre de flottabilité attribuée aux trois bras du tétraèdre est appliquée au simple réseau linéaire.
Bien que cette rigidité soit inférieure à celle espérée, elle est assez importante pour considérer le réseau tétraédrique comme un candidat possible de SDR.
La présente étude a aussi produit un autre résultat important. Nous avons observé que dans les cas de l'un ou l'autre des deux types de réseaux à flottabilité neutre, on peut exprimer la forme des bras des réseaux et leurs décalages maximums à l'aide d'équations simples qui peuvent être évaluées avec une calculatrice de poche.

IMPORTANCE DES RÉSULTATS :

Les présents résultats sont d'une grande utilité pour aider à la conception de systèmes sous-marins à déploiement rapide qui sont actuellement d'une importance cruciale pour les marines. Les programmes informatiques qui permettent de résoudre l'équation générale calculent les formes des câbles de réseaux qui n'ont pas une flottabilité neutre et des équations simples peuvent calculer les formes des câbles à flottabilité neutre, lesquels représentent, en pratique, la plupart des cas. Dans le cas précis du réseau tétraédrique, les calculs de la déformation subie dans un courant ne fournissent qu'un soutien moyen à ce genre de forme.

FUTURS TRAVAUX :

Les travaux relatifs au modèle de démonstration de technologie (DT) du système commun à déploiement rapide (SCDR) seront sous peu en cours au CRDA. De nombreux efforts seront consacrés à la conception de réseaux sous-marins adaptés à l'objectif à atteindre. Les codes machines et les équations simplifiées seront d'une grande utilité pour réaliser ces travaux.

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1.0 Introduction

The Defence Research Establishment Atlantic (DREA) has a requirement for a rapidly deployable hydrophone array that is suitable for matched-field processing. This means, primarily, that the array should have aperture (or extent) in all three physical dimensions. It needs to have length, width and height, with the height being such that the array fills as much of the water column as possible. Moreover, for best results the locations of all the hydrophones in the array need to be known quite accurately at all times, and, preferably, the hydrophones' locations should not be affected very much by the action of a current. Also, in the best of all worlds the array would be inexpensive and capable of rapid deployment.

One suggestion for a simple array that fulfills many of these criteria is that of an array that looks like a tetrahedron. It would consist of three linear arrays joined together and buoyed at their upper ends. Their lower ends would be anchored on the bottom at the three corners of a triangle. The structure gives aperture in all three spatial dimensions, and the geometry, being composed of several triangles, gives the impression of being quite stiff. It looks as if it might not be as vulnerable to current drag as a simple vertical array. However, a counter argument points out that the edges of this structure are not rigid, and they will not even resist a bending moment - let alone compression. Therefore, the advantages of the triangulation are lost.

In order to determine just how well this structure would resist the deforming forces of a current, it was necessary to carry out a detailed calculation. This report describes the study and its results. It compares the deformation of the tetrahedral array to vertical-line arrays that have similar parameters. It also presents very simple equations that adequately describe the array offsets. These equations could be very useful in subsequent design work.

2.0 Analysis of the Array Shape

2.1 Equations Governing the Shape of a Cable in a Current

2.1.1 General Approach

The shape of a cable anchored in a current is described by a set of differential equations that expresses the fact that the forces on every small element of cable are in equilibrium. For a restricted set of conditions, these equations can be solved to give a solution in closed

form; however, the general situation is such that the equations have to be solved numerically. The derivation of the differential equations and the subsequent numerical solution is the subject of this section.

A section of cable hanging in a current of water is subjected primarily to three forces: the cable tension at either end of the piece of cable, the force of gravity on the section of cable, and the drag of the water flowing over it. Although this is true for cable sections of any length, we focus our attention on an infinitesimal cable element for the purpose of deriving the differential equations. Each element (of length dS) is in equilibrium under the action of these three forces. Setting the vector sum of the forces equal to zero will generally result in three differential equations (one for each spatial component), which can then be integrated to produce the shape of the cable. In some instances, of course, the curve of the cable lies in a plane, and only two coordinates (and two differential equations) are involved. The following, however, derives the equations for the more general situation - the one in which all three spatial dimensions are involved. These equations will be needed to analyze the tetrahedral array.

2.1.2 Coordinate System

A set of coordinates particularly useful for this problem is the spherical coordinate system (r, θ, ϕ). (See Figure 1.) At each point on the cable we imagine a unit vector $\hat{\mathbf{r}}$ pointing along the direction of the cable. (In the following, bold characters will represent vectors, and a vector with a caret over it will represent a unit vector.)

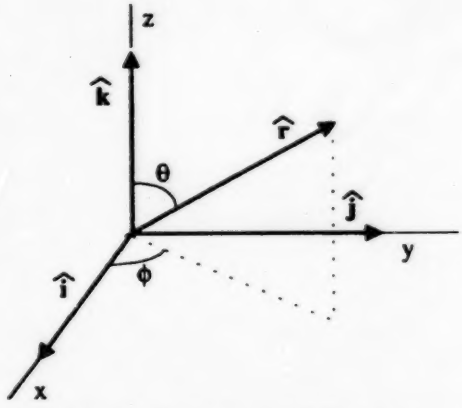

FIGURE 1. This diagram defines the angles θ and φ. It shows the unit vector r, but not the unit vectors $\hat{\theta}$ and $\hat{\phi}$

The set of orthogonal unit vectors $\hat{\theta}$ and $\hat{\phi}$ generally associated with $\hat{\mathbf{r}}$ are not shown in Figure 1 because the diagram would look confused. The unit vector $\hat{\theta}$ is in the direction that $\hat{\mathbf{r}}$ changes as the angle θ increases. Similarly, the unit vector $\hat{\phi}$ is in the direction that $\hat{\mathbf{r}}$ changes as φ increases. The relationship between these unit vectors and the usual cartesian set $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ is as follows:

$$\hat{\mathbf{r}} = \hat{\mathbf{i}} \sin\theta \cos\phi + \hat{\mathbf{j}} \sin\theta \sin\phi + \hat{\mathbf{k}} \cos\theta, \quad (1)$$

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{i}} \cos\theta \cos\phi + \hat{\mathbf{j}} \cos\theta \sin\phi - \hat{\mathbf{k}} \sin\theta, \quad (2)$$

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{i}} \sin\phi + \hat{\mathbf{j}} \cos\phi. \quad (3)$$

The reciprocal relationship is:

$$\hat{\mathbf{i}} = \hat{\mathbf{r}} \sin\theta \cos\phi + \hat{\boldsymbol{\theta}} \cos\theta \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi, \quad (4)$$

$$\hat{\mathbf{j}} = \hat{\mathbf{r}} \sin\theta \sin\phi + \hat{\boldsymbol{\theta}} \cos\theta \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi, \quad (5)$$

$$\hat{\mathbf{k}} = \hat{\mathbf{r}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta. \quad (6)$$

2.1.3 Tension Force

Let us now consider the forces on an element of cable, length dS. First, we will examine the net force on the element due to the tension in the cable itself. A net force on the cable element could be due to a change in the tension as we pass from one end of the element to the other, and it could also be due to a change in direction of the force, which occurs when the cable changes direction.

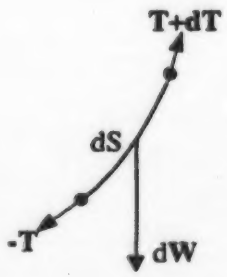

FIGURE 2. This depicts an element of cable with both tension and gravitational forces on it.

Figure 2 shows schematically the element of arclength along with the forces acting at its two ends. The vector force $\mathbf{T}$ can be written as: $\mathbf{T} = T\hat{\mathbf{r}}$ where, again, $\hat{\mathbf{r}}$ is the unit vector lying along the cable at that point. The change in the force ($\mathbf{dT}$) is due to the change in both the magnitude and direction of the force: $\mathbf{dT} = \hat{\mathbf{r}}\, dT + T\,\mathbf{d}\hat{\mathbf{r}}$, and expanding $\mathbf{d}\hat{\mathbf{r}}$ we get:

$$\mathbf{dT} = \hat{\mathbf{r}}\, dT + T\,\mathbf{d}\hat{\mathbf{r}} = \hat{\mathbf{r}}\, dT + T(\hat{\boldsymbol{\theta}}\, d\theta + \sin\theta\, \hat{\boldsymbol{\phi}}\, d\phi), \quad (7)$$

where $d\theta$ and $d\phi$ are the changes in the angles from one end of the arc dS to the other. The force $\mathbf{dT}$ is the net force exerted on the cable element by the tension forces external to the element.

2.1.4 Gravity Force

The weight of the cable element is also depicted in Figure 2. Since it acts vertically downwards, it can be written as: $\mathbf{dW} = -\hat{\mathbf{k}}\, w\, dS$, where 'w' is the weight of the cable per unit length. Using equation 6 to express $\hat{\mathbf{k}}$ in terms of $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$, we obtain:

$$\mathbf{dW} = -w(\hat{\mathbf{r}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta)\, dS \quad (8)$$

2.1.5 Force Due to the Water Current

The hydrodynamic drag force on a cable is usually split into two parts, one component normal (or perpendicular) to the cable, and the other component tangential to the cable. In most situations the normal component is much larger than the tangential component. The normal force is usually taken to be proportional to the square of the normal component of the velocity of water across the cable[1,2,3], and the tangential force is similarly taken to be proportional to the square of the tangential component of the velocity. The equations for the two components of the force usually take the form

$$F_N = .5\,\rho\, v_N^2\, C_N\, D\, L \tag{9}$$

for the normal component and

$$F_T = .5\,\rho\, v_T^2\, C_T\, \pi D\, L \tag{10}$$

for the tangential component, where

ρ is the density of water,
v_N and v_T are the normal and tangential components of the water velocity,
C_N and C_T are the normal and tangential drag coefficients (non-dimensional),
D is the diameter of the cable, and
L is the length of the cable that is under consideration.

Figure 3 shows an element of cable (dS) in a current that is assumed to be flowing in the x-direction. To calculate the normal and tangential force components on this element, we must first express the current in terms of its normal and tangential components. The current can be written as

$$\mathbf{v} = v\,\hat{\mathbf{i}} \tag{11}$$

and, from equation 4, this is

$$\mathbf{v} = v\,(\hat{\mathbf{r}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi) \tag{12}$$

The tangential component of this velocity is the term involving the unit vector $\hat{\mathbf{r}}$, and the normal component of the velocity is the rest of the expression.

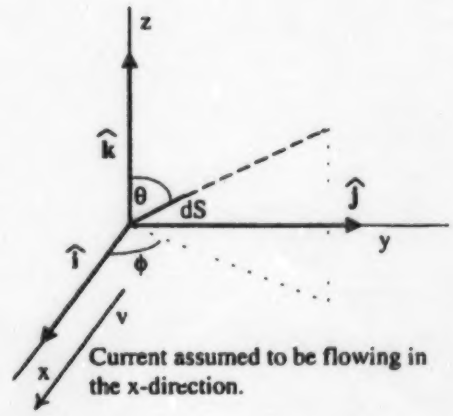

FIGURE 3. This shows an elemental arc-length (dS) in a current of magnitude v flowing in the x-direction.

The magnitude of the drag force is proportional to the square of the velocity, but the tangential component and the normal component must be handled separately. Moreover,

unless the squaring is handled carefully, the proper direction of the force will be lost. The appropriate way to express the squared velocity is to multiply the vector velocity component by its own magnitude. This will maintain the correct directional information while making the magnitude of the force proportional to the velocity squared. Thus, the drag force on the element of cable can be written as:

$$\mathbf{dF_D} = 0.5\rho v|v|\ [C_T \sin\theta\cos\phi|\sin\theta\cos\phi|\ \hat{\mathbf{r}} + C_N\sqrt{\cos^2\theta\cos^2\phi + \sin^2\phi}\ (\cos\theta\cos\phi\ \hat{\theta} - \sin\phi\ \hat{\phi})]\ dS \tag{13}$$

2.1.6 The Differential Equations

Now that we have expressions for all three forces, we can use the fact that they sum to zero to give us the required differential equations. We have the statement of equilibrium:

$$\mathbf{dT + dW + dF_D = 0} \tag{14}$$

This translates into three equations, one for each directional component. By picking out the coefficients of $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\phi}$ from equations 7, 8 and 13 we obtain:

$$dT - w\cos\theta\ dS + .5\rho C_T\ v\sin\theta\cos\phi\ |v\sin\theta\cos\phi|\ dS = 0, \tag{15}$$

$$T d\theta + w\sin\theta dS + .5\rho C_N\ v\ |v|\ \sqrt{\cos^2\theta\cos^2\phi + \sin^2\phi}\ \cos\theta\cos\phi\ dS = 0, \tag{16}$$

$$T\sin\theta\ d\phi - .5\rho C_N\ v\ |v|\ \sqrt{\cos^2\theta\cos^2\phi + \sin^2\phi}\ \sin\phi\ dS = 0. \tag{17}$$

The first can be rewritten to express dT as a function of the arclength dS and the parameters relevant to the problem. The second gives an expression for $d\theta$, and the third gives an expression for $d\phi$. In all cases, the independent variable is the arclength, 'S'. This set of three simultaneous first-order differential equations can be augmented by three more that will provide the cartesian coordinates (x, y, z) of the cable element. These are:

$$dx = \sin\theta\cos\phi\ dS, \tag{18}$$

$$dy = \sin\theta\sin\phi\ dS, \tag{19}$$

$$dz = \cos\theta\ dS. \tag{20}$$

These six simultaneous first-order differential equations can be integrated numerically by techniques such as second-order or fourth-order Runge Kutta. Although faster procedures exist, the second-order Runge Kutta is particularly easy to code up, and, generally, the computing time is so short that it does not pay to spend extra time writing efficient code.

The solution is started at a location where the force on the cable is known - in both magnitude and direction. If, as is assumed here, the cable is suspended by a buoy, the flotation and the current drag of the buoy are used to determine these parameters. In practice, the flotation can be measured, and the drag, which depends on the current, can be calculated by an expression similar to equation 9. In the particular case where the float is spherical, this drag force is

$$F = .5 \rho v^2 C_D \pi r^2 \qquad (21)$$

where πr^2 is the frontal area of the float and C_D is the drag coefficient for a sphere, approximately 0.5.

Once the tension and slope of the top end of the cable are known, the differential equations allow us to step along the cable and calculate the details of its shape and its tension.

Note that in the above discussion we have assumed that the current velocity is a constant, both in magnitude and direction. This is partly a recognition of the fact that details of current versus depth are hard to obtain. And, in practice, it simplifies the algebra and the computer code. However, if the current profile were known in greater detail the modifications to the program would be reasonably straightforward. At each integration step the current would be calculated (from, for example, a table of currents vs. depth), and the resulting drag force would be used in that particular integration step. The difficulty with this is that the depth of the float (and, therefore, the rest of the cable) is not known until after the integration is done, and the integration cannot be done properly until the depths are known. In practice, the calculation would probably require several iterations.

2.2 Application to a Multi-Armed Array

As mentioned above, a single cable anchored at the bottom and buoyed up at the top is quite simple to analyze with the above equation. The buoyancy of the buoy is known, and the current drag can be calculated once the current is known. These two forces can be combined to give the cable's tension and its slope where it attaches to the buoy. The differential equations can then be used to calculate the cable's shape and tension along its entire length.

The situation is more complex when we have more than one cable attached to the buoy. In the case of the tetrahedral array we have three cables meeting at the buoy, each cable having an unknown tension and two unknown angles (θ and ϕ) at the buoy. Unlike the single-cable case, one does not know the initial conditions with which to start the integration of the differential equation.

The nine unknown parameters at the buoy (three tensions and six angles) can be reduced in number to six because there are three equilibrium conditions (one for each spatial dimension) that must be satisfied at the buoy. These six still-unknown parameters can be viewed as eigenvalues of the problem.

The known quantities in this problem are the relative positions of the anchors at the bottom of each of the arms. If (and only if) the six parameters are chosen correctly, the subsequent integrations along the three arms will predict the correct relative displacements of the bottom ends. Our problem is to search for the values of the six parameters that will make the six functional values (the six relative coordinates) correct.

A simplification of the problem can be effected by limiting ourselves to geometries that have a certain degree of symmetry, thereby reducing the number of unknown parameters. The approach we take here is to let the x axis lie in the direction of the current. We place one of the anchors on the x-axis, and we place the other two symmetrically on either side of the axis. The first array arm will lie in the x-z plane and will be parallel to the current. The other two arms are identical, except that they are mirror images of each other about the x-z plane. In this case the six functional values reduce to three - the (vector) displacement of end #2 relative to end #1. (End #3, being the mirror image of end #2, provides no new information.) The corresponding three unknown parameters at the buoy can be taken to be (for example) the tension in one of the side arms and the initial angles (θ, ϕ). Symmetry gives the tension and the two angles of the other side arm, and the two force equations (in the x and z directions) give the tension and the vertical angle associated with the arm that is parallel to the current.

We now have three equations in three unknowns, which is much more tractable than six equations in six unknowns. Moreover, the restriction of the symmetry does not unduly limit the general validity of the results since we are interested primarily in the typical behaviour of the array in a current. The details of an arbitrary array in an arbitrary current are of less interest.

To solve the three equations in three unknowns we use a three-dimensional Newton's method (see, for example, Press[4], Section 9.6). For a given set of parameters (the tension and two angles) we calculate the three functions (errors in the offsets) by integrating down array arms 1 and 2. Also, by making small changes in the parameters one at a time, we calculate approximations to the partial derivatives of all the functions with respect to the unknown parameters. Following the usual Newton's method for multiple equations we invert this matrix of partial derivatives and compute new values of the 'unknown' param-

eters. Newton's method is, however, notoriously unstable if the trial solution is not sufficiently close to the real solution. To prevent this approach from taking us out to no-man's land in solution space, we restrict the maximum distance it can move the trial solution in a single step. We assume that the direction is correct, but we place limits on the magnitude of the step. The procedure is then iterated until the functional values are sufficiently close to zero. This approach has been successful in handling this problem; the algorithm has been stable and has usually generated a solution in less than ten iterations.

3.0 Results

3.1 Shape of Tetrahedral Array in a Current

Figures 4a and 4b show examples of the calculations and plots produced by an IDL routine that has been coded to do the numerical integration and solve for the correct parameters (as described above). Both plots depict the shape of an equi-sided tetrahedron in a current that is running from left to right. The top diagram in each case is the plan view, and the lower diagram is a side elevation. The rope diameter in both cases is 1 cm, and it is assumed to be neutrally buoyant - in other words, the weight term in our differential equation is zero. The drag coefficient has been taken to be 1.2. The float at the top of the tetrahedral array is assumed to be spherical and to have a density of 0.5. A statement of its flotation thus also specifies its radius, a parameter that is used in the drag calculation of equation 21.

The Figure on the left shows an array that is affected only minimally by the current. The array is 50 m high, the flotation at the top is 50 lb (223 N), and the current is 1/4 kt (0.129 m/s). By contrast, the Figure on the right shows an array that is appreciably affected by the current. It is larger (100-metres high), and the current is higher (1 kt). Even though the flotation is four times as large (200 lbs or 890 N) as it is the left-hand case, it is not large enough to hold the array against the current.

One of the interesting things to note in these plots is that the float at the top of the array does not move very much in the current, even in the relatively high current of 1 knot. The upstream cables restrain its motion. On the other hand, the centre of the array bellys out in a shape resembling a catenary.

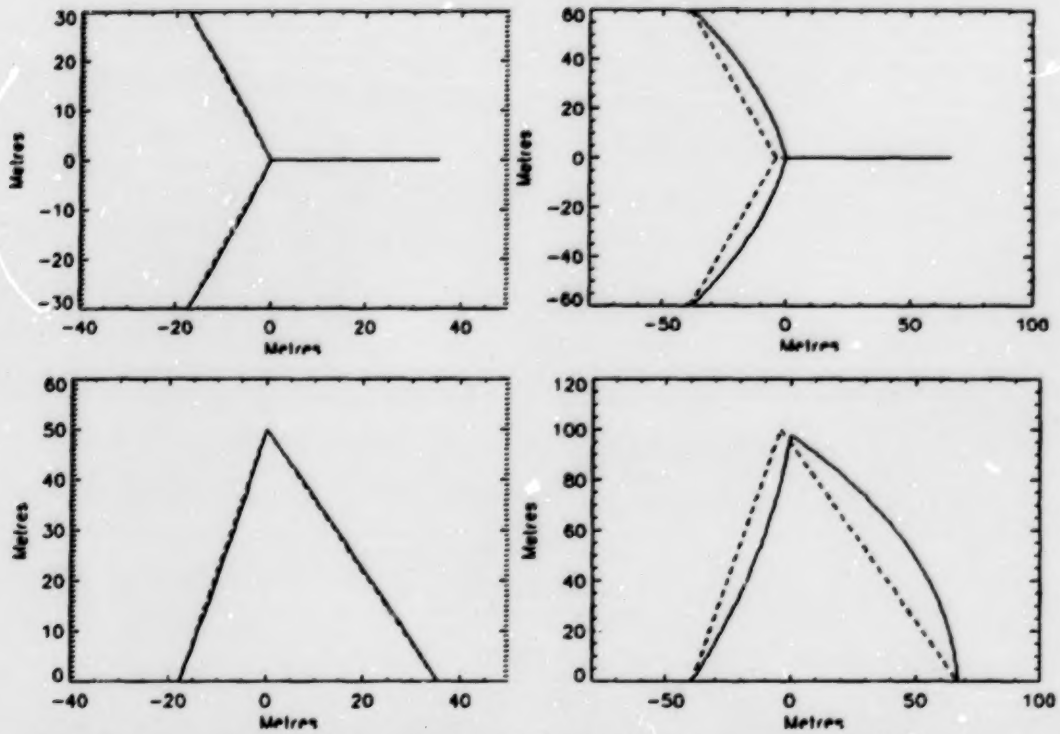

FIGURE 4(a) Plan view and front elevation of an equi-sided tetrahedral array whose height is 50 m. The dashed line represents the array shape in the absence of a current, and the solid line is its shape in a current of 0.129 m/s (1/4 knot). The flotation at the top is 223 N (50 lbs).

FIGURE 4(b) Plan view and front elevation of an equi-sided tetrahedral array whose height is 100 m. The dashed line represents the array shape in the absence of a current, and the solid line is its shape in a current of 0.515 m/s (1 knot). The flotation at the top is 890 N (200 lbs).

3.2 Approximate Offsets When the Array is Neutrally Buoyant

The shape of a weightless cable does more than superficially resemble a catenary; it approximates it very closely for many realistic situations. It can be shown that if the cable is weightless and if the tangential drag (which is generally quite small) is taken to be zero, the cable does, in fact, have the shape of a catenary. The leg of the tetrahedron that is in the (x,z) plane would have the equation:

$$x = \frac{-1}{k}[\cosh(kz)-1] \qquad (22)$$

where the current is assumed to be moving from left to right (in the x-direction). The origin is taken to be at the maximum value of the catenary (i.e., where the tangent line to the function is vertical), which may well be outside the range of the physical cable - as it is in the catenaries in Figure 4. The parameter k is given by:

$$k = .5\rho v^2 D C_N \tag{23}$$

where: ρ is the density of water,
v is the water velocity.
D is the diameter of the cable, and
C_N is the normal drag coefficient.

The other two legs of the tetrahedron also have the shape of a catenary, but they are not in the (x,z) plane, and their mathematical description is a little more complicated. For simplicity in the following discussion we will focus our attention on the leg that is in the (x,z) plane.

If the two end-points of a neutrally-buoyant cable are one above the other and if their separation is L, then the maximum offset due to the current will be at the mid-position (a z-value of L/2). From equation 22 the value of this offset is:

$$\delta = \frac{1}{k}[\cosh(kL/2)-1] = \frac{1}{k}((kL/2)^2/2! + O(kL)^4) \cong kL^2/8 \quad \text{when } kL << 1. \tag{24}$$

Or, writing this in a dimensionless form,

$$\frac{\delta}{L} = \frac{kL}{8}. \tag{25}$$

As discussed before, the tetrahedral leg in the (x,z) plane is a catenary in (x,z) coordinates. However, displacements of the cable in the (x,z) directions are not as interesting or useful as displacements in a direction perpendicular to a line joining the endpoints of the leg. In other words, we would like to work in a coordinate system in which a z'-axis joins the endpoints of the leg and the x'-axis is perpendicular to this. We are interested in displacements in the x'-direction.

Although the mathematical form of the cable shape in the (x', z') coordinate system is not strictly a catenary, the difference is very small for small values of the parameter kL. Figure 5 shows the dimensionless offsets (in the x' direction) calculated (by the IDL program) for a range of parameters.

The dashed line corresponds to the catenary approximation in the sloping coordinate system, and the dotted straight line corresponds to the first-term approximation to the catenary. It has a slope of 1/8 (as does the expression in equation 25). Note that in this plot the 'k' parameter uses the component of water velocity perpendicular to the array line. Note also that the value of 'L' is the length of the array leg - not the height of the array.

This agreement is not accidental. An examination of the differential equation in the skewed set of axes shows that an approximation to a catenary in this coordinate system is to be expected as long as the offset is small.

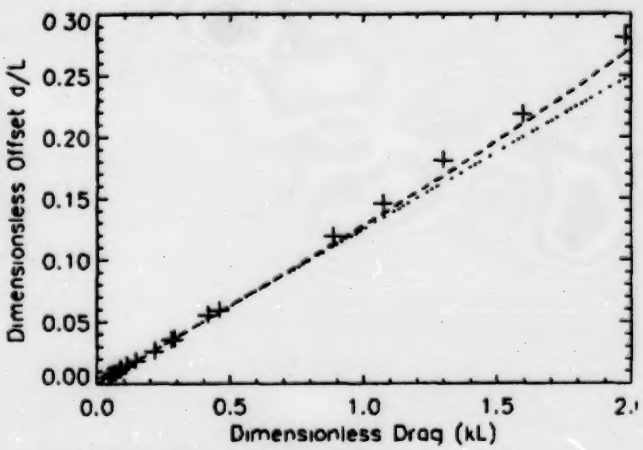

FIGURE 5. This shows the offset of the arm of a tetrahedral array plotted against the dimensionless drag. These computer-generated offsets are measured perpendicularly to the direction of the undeformed arm The dashed line corresponds to the catenary approximation The straight dotted line has a slope of 1/8.

For calculating maximum offsets, therefore, the approximation given by equation 24 can be used with confidence as long as the array is neutrally buoyant and kL is small. In practice, the errors due to incorrect parameters values (the drag coefficient, in particular) will be much larger than errors that stem from this approximation.

3.3 Comparison With Simple Vertical Arrays

A tetrahedral array is heavier, bulkier and much more difficult to install than a simple vertical line array - an array with a weight at one end and a float at the other. Therefore, in order for a tetrahedral array to be preferred over a vertical array, the tetrahedral array must have some very desirable features. It is evident that it does have a finite aperture in all three physical dimensions, which is important for matched-field processing. Also, the computer modelling described above shows that the top of the array is reasonably well anchored; it moves relatively little in a current even though there is a substantial drag force on the float. However, the centres of the tetrahedral legs are pushed sideways by a current, and this could cause difficulties in the analysis of acoustic fields. It is the intent of this section to compare the offsets of tetrahedral legs with that of a vertical array.

Figures 6(a) and 6(b) plot the shape of a vertical array that has roughly the same parameters as the array in Figures 4(a) and 4(b). The height of the array is the same; the diameter of the cable is the same, and the currents are the same. However, the float at the top of each array has one third the flotation of the corresponding float in Figure 4. The purpose of this modification was to make the tensions in the cables roughly the same as those in Figure 4. As can be seen, the maximum offset in Figures 6(a) and 6(b) are substantially larger than the maximum offsets in Figures 4(a) and 4(b).

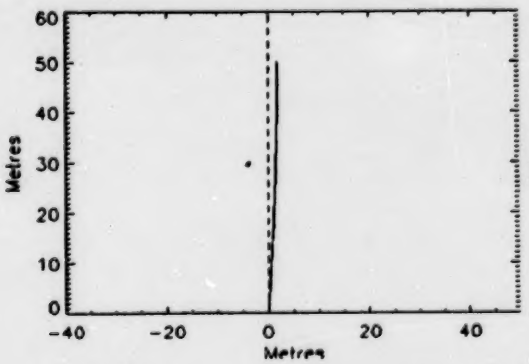

FIGURE 6(a) Elevation view of a vertical array whose length is 50 m. The dashed line is a vertical line for comparison. The solid line is the array shape in a current of 0 129 m/s (1/4 knot). The flotation at the top is 74.3 N (16.7 lbs).

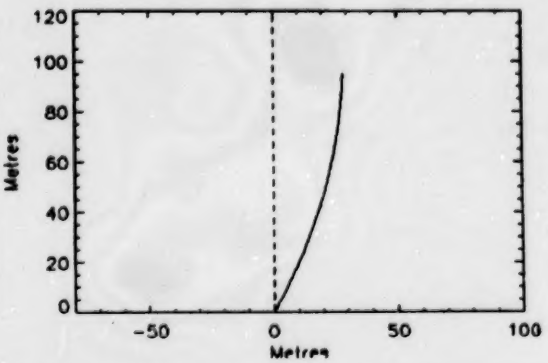

FIGURE 6(b). Elevation view of a vertical array whose length is 100 m The dashed line is a vertical line for comparison. The solid line is the array shape in a current of 0.515 m/s (1 knot). The flotation at the top is 297 N (66.7 lbs)

On the other hand, one can ask whether this reduction in the flotation is fair or realistic. If it is possible, for example, to place 50 lb of flotation in a deployment package for the tetrahedral array, why wouldn't it be possible to put the same flotation in a package for the linear array? The shapes resulting from these higher tensions are shown in Figures 7(a) and 7(b). In this case, the maximum displacements are comparable to, if not less than, the maximum displacements in the tetrahedral arrays (Figure 4). Thus, under one reasonable set of conditions, the displacement of the tetrahedral array is less than that of the vertical array, and under another reasonable set of conditions it is larger. The results depend on what parameters are being held constant.

3.4 Approximations for the Offsets of Vertical Arrays

As was discussed previously, the shape of a cable in a current is a catenary if the cable is neutrally buoyant and if the tangential drag can be ignored. This approximation, plus the fact that the catenary can be treated as a parabola for small offsets gives us the capability of making quick, back-of-the-envelope estimates of the cable displacement.

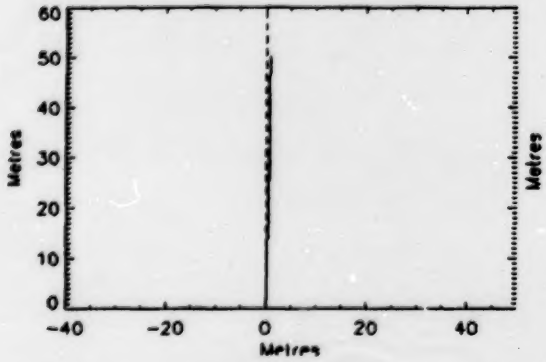

FIGURE 7(a). The parameters for this plot are the same as for Figure 6(a) except that the flotation at the top is the same as for the tetrahedral array in Figure 4(a), namely 223 N (50 lbs).

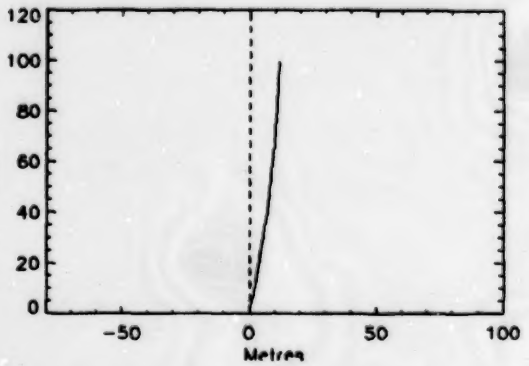

FIGURE 7(b). The parameters for this plot are the same as for Figure 6(b) except that the flotation at the top is the same as for the tetrahedral array in Figure 4(b), namely 890 N (200 lbs).

In the case of the tetrahedral array, the float moved very little, and so the position of the float was not a factor in calculating the array's maximum offset, which occurred at the mid-point of the cable. In the case of the simple vertical array, the maximum displacement is at the float, itself, and so the characteristics of the float play in important part in the simplified expression.

Figure 8 is a diagram of the array buoyed up by the float. The horizontal offset is exaggerated, and we will approximate the height of the float as being equal to 'L', the length of the cable. As in equation 22 the shape of the cable is given by:

$$x = \frac{-1}{k}[\cosh(kz)-1] \quad \text{and, for small 'kz', this is}$$

$$x = -\frac{kz^2}{2} \qquad (26)$$

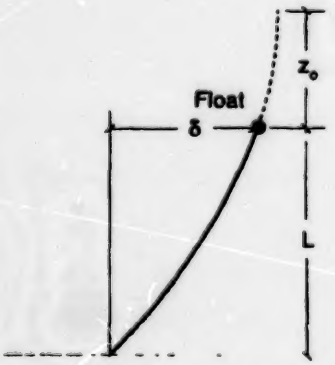

FIGURE 8. Diagram of a neutrally buoyant vertical array in a current

The 'origin' for this equation is the point of maximum 'x', which is above the float in Figure 8. If we can find its distance 'z_o' above the float we will be able to use equation 26 to determine the offset 'δ'. We calculate z_o by using the fact that the reciprocal slope dx/dz is given by the ratio of the drag on the float to its buoyancy. Taking the derivative of equation 26, and solving

for z_o, we obtain (ignoring the sign):

$$z_o = \frac{1}{k}\frac{dx}{dy} = \frac{1}{k}\frac{\text{Drag force}}{\text{Buoyancy}} \tag{27}$$

Using equation 26 we now solve for the horizontal displacement of the buoy:

$$\delta = \frac{k}{2}((L + z_o)^2 - z_o^2) = \frac{k}{2}(L^2 + 2Lz_o) \tag{28}$$

As before (in equation 21), the drag force on the buoy is given by:

$$F = .5\,\rho\, v^2\, C_D\, \pi\, r^2 \tag{29}$$

Thus, equations 27, 28 and 29 allow for a quick calculation of the buoy's offset.

4.0 Conclusions:

This paper addresses itself to the effect of currents on the shape of underwater arrays. The study was motivated by a conjecture that a tetrahedral shaped array might make a very stable structure in water, a structure that, when fitted out with hydrophones, would be suitable for matched-field processing. This structure might be a good candidate for a Rapidly Deployable System (RDS), which, at the moment, is a very important topic in the underwater acoustics community. This study compares the deformations of the tetrahedral array with those of a simple vertical line array.

Unfortunately, these calculations indicate that the tetrahedral array is not substantially more 'rigid' than is a simple vertical line array. It is even less rigid if the buoy's flotation is the same in the two cases, a condition that leads to the tension in each tetrahedral leg being only about one third of that in the vertical array. Thus, although the tetrahedral array has several good features and may well be useful for a Rapidly Deployable System, it will not be chosen on the basis of its 'stiffness' in a current.

In the process of calculating the deformation of the two types of arrays it became evident that if the cables were neutrally buoyant and if the current was 'not too high', the cable shapes could be described by very simple expressions. These equations might be very useful as general design tools.

The paper first shows the derivation of the differential equations describing the shape of an arbitrary cable in a current, and it then shows how this can be used to solve for the effect of currents on a three-armed tetrahedral-shaped array. This procedure, which has been programmed in IDL, is useful for studying the shape of this type of array in a current.

The present program assumes that the current is constant with depth, but this restriction would not be difficult to remove.

Calculations using realistic parameters show that the top of the tetrahedron (where the float is attached) moves very little in a current. The maximum displacement for a neutrally buoyant cable occurs roughly half-way down each leg.

The paper points out that in the neutrally buoyant case the cable takes the form of a catenary, which for small displacements approximates a parabola. With this approximation, the displacement at the centre of the tetrahedral leg is given by the very simple formula:

$$\delta = kL^2/8 , \tag{27}$$

where $k = .5\,\rho\, v_N^2\, C_N\, D$, and

ρ is the density of water,

v_N is the normal component of the water velocity across the array,

C_N is the drag coefficient for flow perpendicular to the cable,

D is the diameter of the cable, and

L is the length of the cable.

Note that 'L' is the slant length of the leg and that 'v_N' is the component of flow normal to this slant direction.

The paper then goes on to discuss vertical line arrays and their deformation due to currents. The maximum displacement for a simple vertical array occurs at the float. When the cable is neutrally buoyant this maximum displacement is given (to good approximation) by:

$$\delta = \frac{k}{2}(L^2 + 2Lz_o) \tag{28}$$

where 'k' and 'L' are as above, and 'z_o' is given by:

$$z_o = \frac{1}{k}\left(\frac{\text{Drag force on buoy}}{\text{Flotation of buoy}}\right) \tag{29}$$

Thus, for both the simple vertical array and the tetrahedral array we have a set of simple equations that allow us to calculate current-driven offsets quickly and reasonably accurately. The only condition is that the legs making up the array be neutrally buoyant in water.

5.0 References:

1. Prandtl, L., and Tietjens, O. G., Applied Hydro- and Aeromechanics. Dover Publications, Inc., New York.

2. Pode, L. Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream. Navy Department, The David W. taylor Model Basin Report S87 NS 830-100, March 1951.

3. Casarella, M. J. and Parsons, M. Cable Systems under Hydrodynamic Loading. Marine Technology Society Journal, vol. 4, n. 4, 1970.

4. Press, W. H., Flannery, B. P., Teukolsky, S. A. and Vetterling, W. T., Numerical Recipes, Published by Cambridge University Press.

List of symbols/abbreviations/acronyms/initialisms

DND	Department of National Defence
DREA	Defence Research Establishment Atlantic
RDS	Rapidly Deployable System
JRDS	Joint Rapidly Deployable System
TD	Technology Demonstrator

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Underwater acoustic arrays shaped like tetrahedrons are possible candidates for rapidly deployable systems (RDS). They have aperture in all three physical dimensions, and therefore might be suitable for matched-field processing. This paper calculates the deformations of such an array in a current and compares this deformation to that of a simple vertical array. The paper derives the equations of the cable shape and presents the results of a number of numerical integrations. It appears that the tetrahedral shapes are not as stiff as one would infer from their triangular nature. The paper then points out very simple equations that will predict the cable shapes under quite common conditions. These back-of-the-envelope equations may well be a very useful design tool

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